

Effect of Bond Layer on Tri-Layered Assembly Subjected To Differential Uniform Temperature Change

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ABSTRACT

In the present analytical and numerical study, the thermal mismatching stress induced under differential temperature conditions of tri-layered assembly with bond is investigated. The thermal mismatching stresses are one of the reasons for structural failures between two or more connected devices. Therefore it is very essential to understand variation of these stresses and estimation in the interfaces play an important role in design and reliability studies of microelectronic assemblies. In this paper, a physical model is proposed for the interfacial shearing and peeling stresses occurring at the interfaces of tri-layered dissimilar materials with the effect of bonding subjected to differential uniform temperature in the layer. It observed from both analytical and numerical study that the shearing stress reduced in the range of 60% to 70% at interface (1-2) and 35% to 40% at (2-3) interface. Peeling stress are continuously reduced in the range of 10% - 20% at (1-2) interface and 13% - 35% at (2-3) interface due to the influence of bond layer. Thus, it indicates that, the bond layer consideration may influence significantly on interfacial stress. It is found that the both interfacial shearing stresses and peeling stresses decreased considerably at the interface with the increase of bond layer thickness.

Keywords – Tri-layered model, Shearing stress, Peeling stress, Different uniform temperature model, Electronic package

I. INTRODUCTION

Thermo mechanical mismatch induced interfacial stresses of the major concerned in the structural failure between two or more connected devices. The electronic assemblies are the heat generating source as they operated under high power conditions therefore thermal mismatching stresses are inevitably arises in the interfaces of bonded dissimilar metals, this is because of the differences in the co-efficient of thermal expansion. Timoshenko's [1] proposed the fundamental solution to thermal stresses of biomaterial using a beam theory. The electronic signals may be incorrectly transferred and that may cause structural failure [2]. When two thin plates are bonded together, an extremely thin bond layer of third material is exists between them. Adhesively bonded and soldered bi-material assemblies are widely used in micro- and Opto-electronics [3-5].

Since from last two decades, the research on the thermal stresses on the structured layered is carried out. The effect of the bond layer is inspired by derivations of Chen and Nelson [6] and suhir considering the interfacial shearing and peeling stresses in to account. In the recent years many other researches Mirman [7], Matthys and Mey [8], Ru [9] and Moore and Jarvis [10-11] contributed on this aspect. An improved bi-material uniform temperature

model accounting for differential uniform temperature and thickness wise linear temperature gradient in the layers have been carried out by sujan et al. [12].

The model of tri-layered assembly subjected to uniform temperature was first proposed by Schmidt [13] in 1999 and suhir [14] in 2003. The mathematical model formed by these two authors is inconsistencies in consideration of the exponential parameter k in the interfacial shearing stress expression. This inconsistency is well addressed by sujan et al. [15] and proposed improved tri-material solution for both interfacial shearing and peeling stresses. It is found that the numerical solution suggested close agreement compare to earlier models. The bond layer acts as interfacial shear stress compliance between the two principal layers. Consequently, it will have some influence on the interfacial stresses in a Bi-material assembly. The value of interfacial shear stress compliance for the bond layer at the interface was proposed by Sujan [16] which is given as K_0 as h_b/G_0 . A Gold-Tin solder bond is introduced as the bond layer between silicon and diamond layers and they show that the effect of bond layer on the interfacial shearing and peeling stress. Recently, Sujan et al. [17] studied the tri-layered interfacial stress model with the effect of different temperatures in the layers only. The effect

of bond layer on interfacial shear and peeling stresses is not considered. It is observed that the effect of linear temperature gradient may influence interfacial stresses considerably. Recently, Aswath et al. [18] has been studied the effect of bond thickness on tri-layered assembly subjected to uniform temperature effect. It is found that the interfacial shearing and peeling stresses are decreased considerably at the interface with the increase of bond thickness.

However, to date no attempt as been made to study the effect of the thermal mismatching interfacial stresses and the tri-material assembly with bond subjected to differentially uniform temperature change. The thermo mechanical stresses as significance to understanding of the nature of interfacial stresses under different temperature conditions is necessary to minimize or eliminate the risk of structural failure.

The aim of the article is to present shear compliance expressions to account for bond layer effect. The effect of bond layer on interfacial shearing and peeling stress models subjected to differential uniform temperatures was presented.

II. MATHEMATICAL FORMULATIONS

a. Analytical Method

Fig. 1 shows the physical model of full length (2L) of the tri-layered assembly with the three layers designated as 1, 2 and 3 and a free body diagram for a cut at some arbitrary x location. E, α, ν, and h represent elastic modulus, thermal expansion coefficient, Poisson's, and thickness of i-th layer and ΔT differential uniform temperature change in the layers.

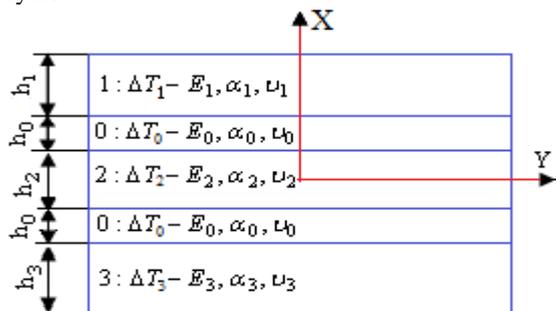


Fig. 1 Physical and materials properties of tri-layered assembly with bond layer at the interface.

To develop the analytical model of shearing stress Sujun [15] is referred until basic governing equations. The forces F₁ and F₂ at any section of the layers in Fig.1 are given by,

$$F_1 = \int_{-L}^x \tau_1 dx \text{ and } F_2 = \int_{-L}^x \tau_2 dx \quad (1)$$

Where, τ₁ and τ₂ represent shear stress between top-middle and middle-bottom layers respectively.

Considering the bonding layer effect, the compatibility condition at the interfaces are expressed as,

$$\left. \begin{aligned} \epsilon_{x(1)}^B + \epsilon_{x(2)}^T &= K_0 \frac{\partial \tau}{\partial x} \\ \epsilon_{x(2)}^B + \epsilon_{x(3)}^T &= K_0 \frac{\partial \tau}{\partial x} \end{aligned} \right\} \quad (2)$$

Where, ε_{x(i)}, i = 1, 2, 3 are axial strains given by

$$\epsilon_{x(i)} = \frac{\partial U_i}{\partial x} \text{ and } U_i \text{ represents the axial displacement of the } i\text{-th layer and superscripts B and T denote bottom and top surfaces.}$$

Considering moment equilibrium about positive Z-axis (perpendicular to the paper upward) at x and y = 0 is given by,

$$\left. \begin{aligned} M_1 + M_2 + M_3 - \frac{1}{2}(h_1 + h_2 + h_0)F_1 \\ - \frac{1}{2}(h_2 + h_3 + h_0)F_2 = 0 \end{aligned} \right\} \quad (3)$$

Since, M_i = $\frac{D_i}{R}$, the expression for radius of curvature is

$$\frac{1}{R} = \left(\frac{h_1 + h_2 + h_0}{2D} \right) F_1 + \left(\frac{h_2 + h_3 + h_0}{2D} \right) F_2 \quad (4)$$

In equation (4) D = D₁ + D₂ + D₃

$$\text{Where, } D_i = \frac{E_i h_i^3}{12(1 - \nu_i^2)}, i = 1, 2, 3$$

Axial strains at the interfaces of the uniformly heated three layered structure take the form,

$$\left. \begin{aligned} \epsilon_{x(1)}^B &= \alpha_1 \Delta T_1 + \lambda_1 F_1 + \frac{h_1}{2R} - K_1 \frac{\partial \tau_1}{\partial x} \\ \epsilon_{x(2)}^T &= \alpha_2 \Delta T_2 + \lambda_2 (F_2 - F_1) - \frac{h_2}{2R} + K_2 \frac{\partial \tau_1}{\partial x} \\ \epsilon_{x(2)}^B &= \alpha_2 \Delta T_2 + \lambda_2 (F_2 - F_1) + \frac{h_2}{2R} - K_2 \frac{\partial \tau_2}{\partial x} \\ \epsilon_{x(3)}^B &= \alpha_3 \Delta T_3 + \lambda_3 F_2 - \frac{h_3}{2R} + K_3 \frac{\partial \tau_2}{\partial x} \end{aligned} \right\} \quad (5)$$

Where, axial compliance, λ_i = $\frac{(1 - \nu_i^2)}{E_i h_i}$, coefficient of interfacial compliance, K_i = $\frac{h_i}{3G_i}$. Substituting

(5) into (2), produces

$$\left. \begin{aligned} \Delta T_1 (\alpha_1 - \alpha_2) + \lambda_{12} F_1 - \lambda_{20} F_2 - K_{12} \frac{\partial \tau_1}{\partial x} &= \frac{K_0 \partial \tau}{\partial x} \\ \Delta T_2 (\alpha_2 - \alpha_3) + \lambda_{23} F_2 - \lambda_{20} F_1 - K_{23} \frac{\partial \tau_2}{\partial x} &= \frac{K_0 \partial \tau}{\partial x} \end{aligned} \right\} \quad (6)$$

Where, K_{ij0} = K_i + K_j + K₀,

$$\lambda_{20} = \lambda_2 - \frac{(h_1 + h_2)(h_2 + h_3)}{4D}$$

$$\lambda_{ij} = \lambda_i + \lambda_j + \frac{(h_1 + h_2)^2}{4D}$$

Shear compliance for bond layer, $K_0 = h_0/G_0$.
 The solution for equation (6) is assumed to be of the form:

$$\tau_i = A_i^{(1)} \sinh(k_1 x) + A_i^{(2)} \sinh(k_2 x) \quad (7)$$

Where, $A_i^{(1)}$ and $A_i^{(2)}$ are arbitrary constants for $i = 1, 2$ and k_1 and k_2 are roots of certain characteristic equation.

Where,

$$\left. \begin{aligned} A_1^{(1)} &= \frac{\Delta T_1 [(\alpha_1 - \alpha_2)\beta_2 + (\alpha_2 - \alpha_3)\gamma]}{k_1 K_{120} (k_2^2 - k_1^2) \cosh(k_1 L)} \\ A_1^{(2)} &= \frac{\Delta T_2 [(\alpha_1 - \alpha_2)\beta_1 - (\alpha_2 - \alpha_3)\gamma]}{k_2 K_{120} (k_2^2 - k_1^2) \cosh(k_1 L)} \\ A_2^{(1)} &= \beta_3 \frac{\Delta T_1 [(\alpha_1 - \alpha_2)\beta_2 + (\alpha_2 - \alpha_3)\gamma]}{k_1 K_{120} (k_2^2 - k_1^2) \cosh(k_1 L)} \\ A_2^{(2)} &= \beta_4 \frac{\Delta T_2 [(\alpha_1 - \alpha_2)\beta_1 - (\alpha_2 - \alpha_3)\gamma]}{k_2 K_{120} (k_2^2 - k_1^2) \cosh(k_1 L)} \end{aligned} \right\} \quad (8)$$

Where,

$$\beta_1 = \frac{(\lambda_{12} - k_1^2 K_{120})}{K_{120}}, \beta_2 = \frac{(-\lambda_{12} + k_2^2 K_{120})}{K_{120}},$$

$$\beta_3 = \frac{(\lambda_{12} - k_1^2 K_{120})}{K_{20}}, \beta_4 = \frac{(\lambda_{12} - k_2^2 K_{120})}{K_{20}} \text{ and}$$

$$\gamma = \frac{\lambda_{20}}{K_{220}}$$

Characteristic equation:

$$k_i^2 = \frac{\left[r \pm (r^2 - 4K_{120}K_{230}S)^{1/2} \right]}{2K_{120}K_{230}} \quad (9)$$

Where, $r = \lambda_{12}K_{230} + \lambda_{23}K_{120}$.

$$S = \lambda_{12}\lambda_{23} - (\lambda_{20})^2$$

Now, shearing stress τ_1 and τ_2 at interfaces are determined by using equation (7). Peeling stress at the interfaces is given by:

$$P_1 = - \left[a_1 \frac{\partial \tau_1}{\partial x} + a_2 \frac{\partial \tau_2}{\partial x} \right] \quad (10)$$

$$P_2 = \left[b_1 \frac{\partial \tau_1}{\partial x} + b_2 \frac{\partial \tau_2}{\partial x} \right] \quad (11)$$

Where, $a_1 = \frac{D_1 h_1 + D_1 h_2 - D h_1}{2D}$,

$$a_2 = \frac{D_1 (h_2 + h_3)}{2D}, b_1 = \frac{D_3 (h_1 + h_2)}{2D} \text{ and}$$

$$b_2 = \frac{D_3 h_2 + D_3 h_3 - D h_3}{2D}$$

$$\frac{\partial \tau_1}{\partial x} = A_1^{(1)} k_1 \cosh(k_1 x) + A_1^{(2)} k_2 \cosh(k_2 x)$$

$$\frac{\partial \tau_2}{\partial x} = A_2^{(1)} k_1 \cosh(k_1 x) + A_2^{(2)} k_2 \cosh(k_2 x)$$

III. NUMERICAL INVESTIGATIONS

a. Verification OF ANALYTICAL RESULTS

The grid independent study has been made with different grids to yield consistent values of Sujan et al. [15]. In the present numerical investigation the SOLID 45 element is used for the 3D modeling of solid structures. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. However, element SLOID 45 is selected for the analysis of the interfacial stresses between bi-layered and tri-layered assembly because of the element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities.

During the course of present research, the present methodology is verified in terms of interfacial stresses like shearing stresses and peeling stresses, In order to validate the predictive capability and accuracy of the present methodology, computations are performed using the configuration and boundary conditions of the analytical and numerical investigation by Sujan [15] on effects tri-material assembly without bonding is selected. The results presented in the paper in terms of shearing and peeling stresses for without bonding material between interfacial materials. It is seen from the literature that, Sujan [15] used SOLID45 elements with (7000 elements) constant mesh for ¼ of the total model due to double symmetrical model. The authors have made assumptions during the analysis of both analytical and numerical investigations which were explained in part

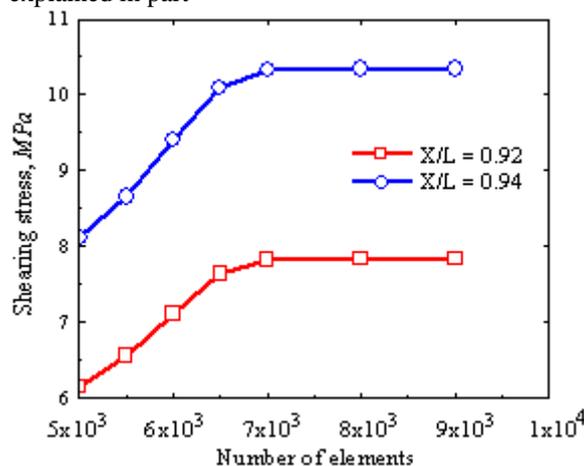


Fig. 2 Convergence of shearing stresses with grid refinement.

Since the system is double symmetric, for 3D analysis one quarter of the model is analyzed. For 3D

model, Sujan [15] used for layer 1: $10 \times 10 \times 5 + 40 \times 10 \times 5 = 2500$, for layer 2: $10 \times 10 \times 3 + 40 \times 10 \times 3 = 1500$ and $10 \times 10 \times 6 + 40 \times 10 \times 6 = 3000$ elements for layer 3 (total number of elements = 7000). The BRICK 8 noded SOLID 45 element with uniform grid is selected for the numerical analysis. Different grid sizes with total number of elements 5000, 6000, 7000, 8000 and 9000 constant mesh have been studied. The grid with 7000 elements gave results identical to that of 8000 and above shown in Fig. 2. In view of this, 7000 elements grid is used in all further computations. Fig. 3 shows the one quarter of 3D model for the numerical analysis. It may be noted that Sujan [15] have used a constant mesh of 7000 for their study.

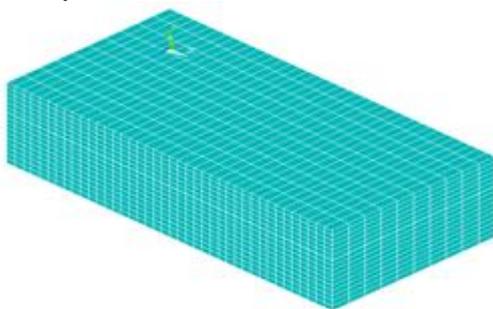


Fig. 3 One quarter of 3D model after mapped meshing.

Table 1 Compression of shearing stress of previous work with present work for uniform temperature at (1-2) and (2-3) interfaces.

x/L	Interface (1-2)		Interface (2-3)	
	Sujan [15]	Present study	Sujan [15]	Present study
0.9	6.60	6.45	-5	-4.98
0.92	7.99	7.82	-4.43	-4.39
0.94	10.42	10.32	-5.61	-5.55
0.96	13.66	13.74	-8.18	-8.09
0.98	17.99	18.15	-9.69	-9.59
1	23.82	24.54	-8.23	-8.1

A comparison of the interfacial shear stress for layers 1-2 and 2-3 are made with Sujan et al. [15]. Table 1 and 2 show the comparison of interfacial shear stresses of layer 1-2 and 2-3 subjected to constant temperature respectively. It is observed from table 1 that there is a good agreement between the present results and that of Sujan et al. [15] for interfaces of 1-2 and 2-3 with maximum discrepancy of 2.2%.

IV. RESULTS AND DISCUSSION

The tri-layered assembly with bond subjected to differential temperature at the interfaces is shown in Fig. 1. The analytical computations are carried out for the thickness of the bond varied from 0.001 to 0.004mm in order to find the variations of the

interfacial shear and peeling stresses.

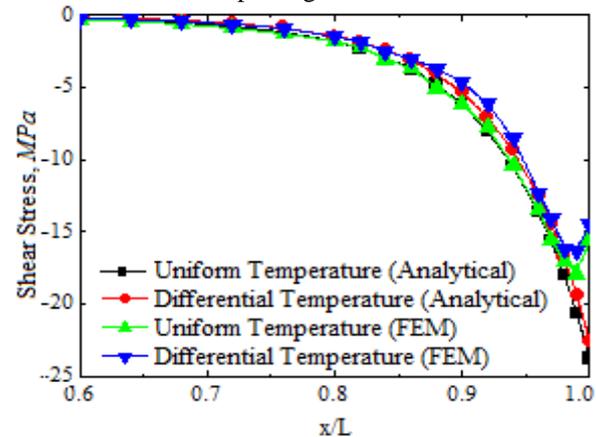


Fig. 4.1 Shearing stress along the interface of layer (1-2) for uniform temperature model, ($\Delta T = -120^\circ\text{C}$) and differential uniform temperature model, ($\Delta T_1 = -120^\circ\text{C}$, $\Delta T_2 = -120^\circ\text{C}$, and $\Delta T_3 = -60^\circ\text{C}$).

Fig. 4.1 represents shearing stress along the interface of layer (1-2) for the cases of uniform temperature change (UTC), and differential uniform temperature change (DUTC). From the comparison of analytical values between UTC and DUTC, it can be seen that for DUTC, shearing stress is considerably lower compared to UTC at all identical points along the interface of layer (1-2). However, analytical comparison shows that for DUTC, shearing stress reduces almost 18% in average compared to UTC. The numerical (FEM) simulation is also represented in Fig. 4.1. The variations of interfacial shearing stresses are similar to that of analytical one. However, FEM comparisons shows that for DUTC the shearing stress reduced almost 20% in average at the interface 1-2 compared to UTC. Thus, it is observed from the Fig. 4.1 that there is a good agreement between the analytical and numerical simulations with maximum discrepancy of 3%.

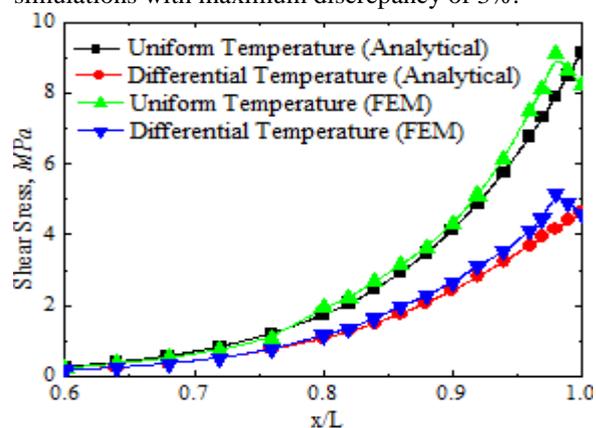


Fig. 4.2 Shearing stress along the interface of layer (2-3) for uniform temperature model, ($\Delta T = -120^\circ\text{C}$) and differential uniform temperature model, ($\Delta T_1 = -120^\circ\text{C}$, $\Delta T_2 = -120^\circ\text{C}$, and $\Delta T_3 = -60^\circ\text{C}$).

Fig. 4.2 represents the analytical and numerical simulations of shearing and peeling stresses along the interface of layer (2-3) for the cases of UTC and DUTC. Analytical comparison between these two shows that for DUTC, shearing stress is substantially lower compared to UTC at all identical positions along the interface of layer (2-3). The shear stress is increased monotonically with increase of length. For instance, at location $x/L = 0.80$, for DUTC shearing stress is lower by 0.65 MPa compared to UTC, at $x/L = 0.9$ the value increases to 1.67 MPa, and at the free end ($x/L = 1$), the difference increases as much as almost 50%. However, analytical comparison shows that for DUTC, shearing stress reduces almost 40% in average at the interface layer (2-3) compared to UTC. However, the FEM comparison shows that for DUTC, shearing stress reduces almost 38.5% in average at the interface of layer (2-3) compared to UTC. Thus, again almost similar trend of variation is reflected for analytical and numerically simulated results.

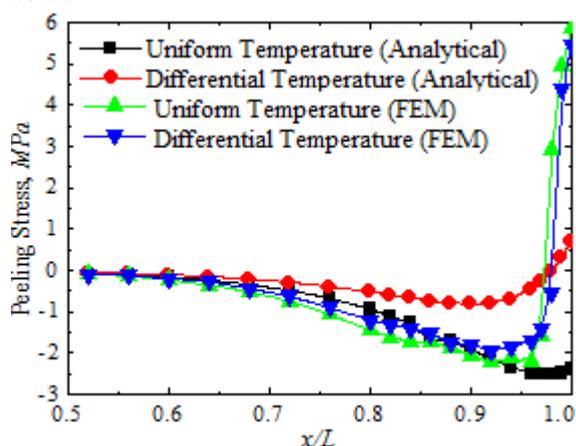


Fig. 4.3 Peeling stress along the interface of layer (1-2) for uniform temperature model, ($\Delta T = -120^\circ\text{C}$) and differential uniform temperature model, ($\Delta T_1 = -120^\circ\text{C}$, $\Delta T_2 = -120^\circ\text{C}$, and $\Delta T_3 = -60^\circ\text{C}$).

The analytical and numerical simulations of peeling stresses of interfacial layers (1-2) subjected to UTC and DUTC is shown in Fig. 4.3. It is observed that the analytical comparison shows that the peeling stresses for DUTC considerably lower compared to UTC at all identical locations along the interface beyond $x/L > 0.6$. However, analytical comparison shows that for peeling stress reduces almost 60% in average at the interface of layer 1-2 compared to analytical with exception of 3 locations near the free end and represent FEM simulation for respectively.

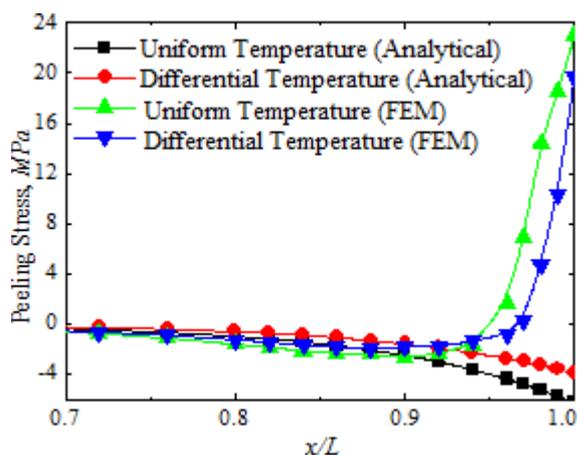


Fig. 4.4 Peeling Stress along the Interface of Layer (2-3) for Uniform Temperature Model, ($\Delta T = -120^\circ\text{C}$) and Differential Uniform Temperature Model, ($\Delta T_1 = -120^\circ\text{C}$, $\Delta T_2 = -120^\circ\text{C}$, and $\Delta T_3 = -60^\circ\text{C}$)

However, analytical comparison shows that for analytical, peeling stress reduces almost 35% in average at the interface of layer 2 and layer 3 compared to FEM respectively. The comparison between the two graphs shows that at location $x/L = 0.8$. However, FEM comparison shows that peeling stress reduces almost 30% in average at the interface of layer 2 and layer 3 compared to analytical. Although beyond $x/L = 0.96$ till the free end the peeling stress changes sign as was observed earlier.

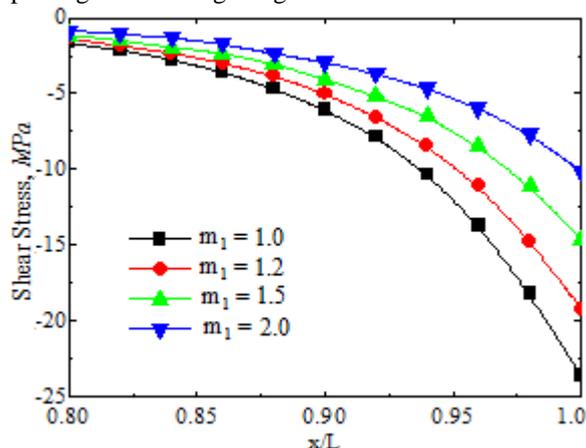


Fig. 4.5 Shearing stress along the interface of layer (1-2) with temperature ratio (m_1).

Fig. 4.5 shows analytical comparison of shearing stress along the interface of layer 1-2 for different values of m_1 with $m_2 = 1$. It is observed that at $x/L = 0.8$, for $m_1 = 2$ shearing stress reduces by 0.8 MPa compared to $m_1 = 1$. At $x/L = 0.9$ the difference increases to 4.14 MPa and at $x/L = 1$, the difference increases as much as 14.96 MPa or 60%

The variation of shearing stress along the interface of layer 2-3 for different values of m_1 with

$m_2 = 1$. It can be observed that at $x/L = 0.8$, for $m_1 = 2$ shearing stress reduces by 0.48 MPa compared to $m_1 = 1$. At $x/L = 0.9$, the difference increases to 2.03 MPa and at $x/L = 1$, the difference further increases to 2.59 MPa or 20%.

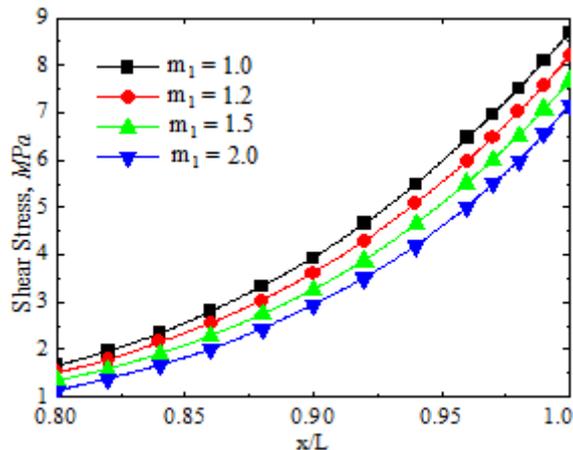


Fig. 4.6 Shearing stress along the interface of layer (2-3) with temperature ratio (m_1).

Fig. 4.6 represents shearing and peeling stresses based on various values of m_1 where m_2 is maintained constant. Here α_i is varied from $16 \times 10^{-6} / ^\circ\text{C}$ to $3.2 \times 10^{-6} / ^\circ\text{C}$ in four stages to produce the value of $n_1 = 0.2, 0.3, 0.4,$ and 0.5 maintaining $m_2 (= \alpha_2/\alpha_3)$ constant. Thus, again almost similar trend of variation is reflected for analytical and numerically simulated results.

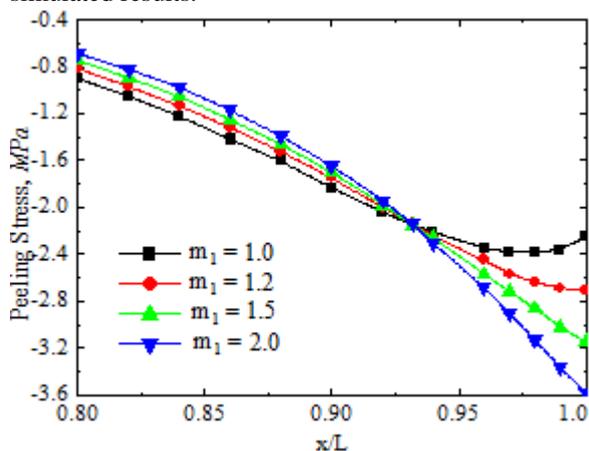


Fig. 4.7 Peeling stress along the interface of layer (1-2) with temperature ratio (m_1).

Fig. 4.7 shows analytical comparison of peeling stress for different values of m_1 with $m_2 = 1$. For the combination of the layers in this case, along the interface of layer 1 and layer 2, the peeling stress is compressive in nature. It can be observed that at $x/L = 0.8$, for $m_1 = 2$ peeling stress reduces by 0.22 MPa compared to $m_1 = 1$. From $x/L = 0.8$ the difference starts decreasing until location $x/L = 0.94$ where the stress values are almost identical. From x/L

$= 0.94$ towards the free end we can observe a reverse trend where peeling stress value for $m_1 = 2$ starts increasing compared to $m_1 = 1$ and at $x/L = 1$, the difference increases to 2.39 MPa or 62%.

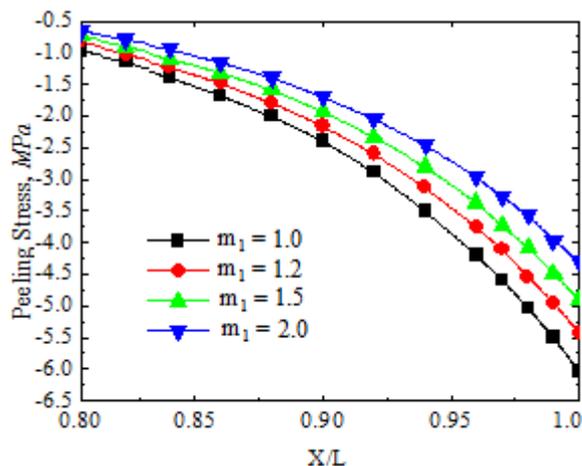


Fig. 4.8 Peeling stress along the interface of layer (2-3) with temperature ratio (m_1).

Fig. 4.8 shows the variation of peeling stress along the interface of layer 2 and layer 3 for different values of m_1 with $m_2 = 1$ is shown in Fig. 4.8. For the combination of the layers in this case, the peeling stress is compressive in nature. It can be observed that at $x/L = 0.8$, for $m_1 = 2$ peeling stress reduces by 0.30 MPa compared to $m_1 = 1$. At $x/L = 0.9$ the difference increases to 0.75 MPa and at $x/L = 1$, the difference further increases to 1.95 MPa. However, it is found that for $m_1 = 2$, peeling stress reduces almost 30% compared to $m_1 = 1$ at any identical location at that interface. Thus, it is evident that the different levels of temperatures in the layers has significant influence in the shearing and peeling stress development and should be accounted while calculating interfacial stresses in a tri-material assembly.

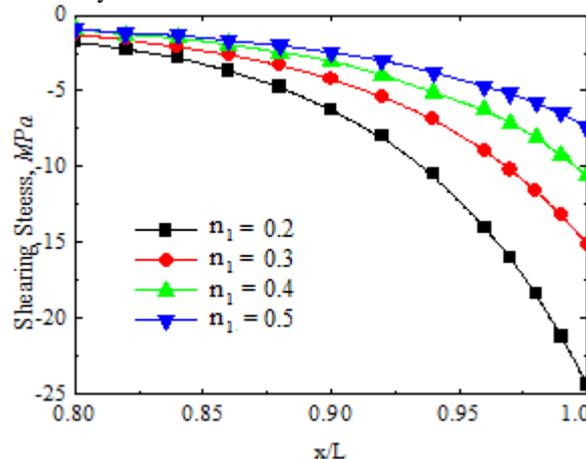


Fig. 4.9 Shearing stress along the interface of layer (1-2) with coefficient of thermal expansion ratio (n_1).

Fig. 4.9 shows analytical comparison of shearing stress along the interface of layer (1-2) for different values of n_1 where n_2 is maintained constant. It can be observed that with increasing the value of n_1 , shearing stress decreases considerably at the interface. For instance, at location $x/L = 0.8$ for $n_1 = 0.5$ shearing stress reduces by 1.03 MPa compared to $n_1 = 0.2$. At $x/L=0.9$, the difference increases to 4.03 MPa and at $x/L = 1$, the difference further increases as much as 18.86 MPa or 73%.

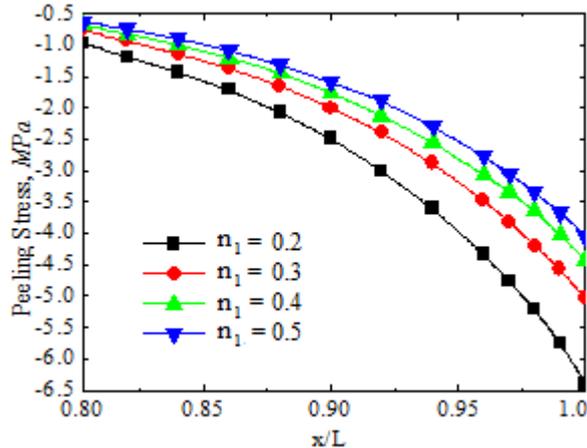


Fig. 4.10 Peeling stress along the interface of layer (2-3) with coefficient of thermal expansion ratio (n_1).

Fig 4.10 shows that The variation of peeling stress along the interface of layer 2 and layer 3 for different values of n_1 with constant n_2 is shown in Fig. 4.12. For the combination of the layers, the peeling stress is compressive in nature in this case. It is observed that at $x/L = 0.8$, for $n_1 = 0.5$ peeling stress reduces by 0.38 MPa compared to $n_1 = 0.2$. At $x/L = 0.9$ the difference increases to 0.96 MPa and at $x/L = 1$, the difference further increases to 2.32 MPa or 35%. However, it is found that for $n_1=0.5$, peeling stress reduces by around 36% in average compared to $n_1=0.2$ at any identical location at that interface.

V. CONCLUSION

Thorough validation of both analytical and numerical analysis is carried out for both the shearing and peeling stress. The results obtained from the analysis leads to following conclusion.

1. Analytical and Numerical results showed that shearing stress are reduced in the range of 60% - 70% at (1-2) interface and 35% - 40% at (2-3) interface near the free end due to the influence of bond layer. Thus, it indicates that near the vicinity of the free end, the bond layer consideration may influence significantly on interfacial stress.
2. It is observed that, peeling stress are continuously reduced in the range of 10% - 20% at (1-2) interface and 13% - 25% at (2-3) interface due to the influence of bond layer. Thus, it indicates that,

the bond layer consideration may influence significantly on interfacial stress.

3. The shearing stresses decreased considerably at the interface with the increase of bond layer thickness. For instance, shearing stress decreased 40% - 50% at (1-2) interface and 25% - 40% at (2-3) interface respectively at the free end for a bond thickness of 0.01mm compared to zero bond thickness.
4. The peeling stresses decreased considerably at the interface with the increase of bond layer thickness. For instance, peeling stress decreased by 14% - 20% at both (1-2) interface and (2-3) interface respectively at the free end for a bond thickness of 0.01mm compared to zero bond thickness.

REFERENCES

- [1] Timoshenko, *Analysis of Bi-metal Thermostats*, *J. opt . Soc. Am.* 11, 1925, 233-255.
- [2] M Vujosevic, *Thermally Induced Deformations in Die-substrate Assembly*, *Theor. Appl. Mech.*, 35(1-3), 2008, 305-322.
- [3] W T Chen and C W Nelson, *Thermal Stresses in Bonded Joints*, *IBM Journal, Research and Development*, vol.23, No.2, 1979.
- [4] F V Chang, *Thermal Contact Stresses of Bi-Metal Strip Thermostat*, *Applied Mathematics and Mechanics*, vol.4, No.3, Tsing-hua Univ., Beijing, China, 1983.
- [5] E. Suhir, *Stresses in Bi-Metal Thermostats*, *ASME Journal of Applied Mechanics*, vol. 53, No. 3, Sept. 1986.
- [6] W T Chen and C W Nelson *Thermal Stresses in Bolted Joints*, *IBM J. Res. Dev.*, 23, 1979, 178-188.
- [7] I B Mirman, *Effect of Peeling Stresses in Bi-Material Assembly*, *ASME J. Electron. Packag.*, 11, 1991, 431-433.
- [8] L Matthys and G D Mey, *An analysis of an Engineering Model for the Thermal Mismatch Stresses at the Interface of a Uniformly Heated two Layer Structure*, *Int. J. Electron. Packag.*, 19(3), 1996, 323-329.
- [9] C Q Ru, *Interfacial Thermal stresses Bi-material Elastic Beams: Modified Beam Models Revisited*, *ASME J. Electron. Packag.* 124(3), 2002, 141-146.
- [10] T D Moore and J L Jarvis, *A Simple and Fundamental Design Rule for Resisting Delamination in Bi-material Structures*, *Microelectron. Reliab.*, 43, 2003, 487-494.

- [11] T D Moore and J L Jarvis, *The Peeling moment-A Key Rule for Delamination Resistance in I.C. Assemblies*, ASME J. Electron. Packag. 126, 2004, 106-109.
- [12] D Sujan, K N Seetharamu, A Y Hassan and M V V Murthy, *Engineering Model for Interfacial Stresses of a Heated Bi-material Structure Used in Electronic Packaging*, Int. Conf. EMAP, Penang, Malaysia, 2004, 191-197.
- [13] W. D. Brown, *Advanced Electronic Packaging* (IEEE Press, New York, 1999, 241-266).
- [14] E Suhir, *Analysis of Interfacial Thermal Stresses in a Tri-material Assembly*. J. Appl. Phys. 89(7), 2001, 3685–3694.
- [15] D. Sujan, M. V. V. Murthy, K. N. Seetharamu, *An improved closed-form Solution for shearing and peeling stresses of tri-material assembly in electronic packaging*. Int. Microelectron. Packag. Soc. JMEP 5(1), 2008, 37-42.
- [16] D. Sujan, W. E. Dereje, M. V. V. Murthy and K. N. Seetharamu, “*Effect of bond layer on Bi-Material Assembly subjected to Uniform Temperature Change*” ASME Journal of Electronic Packaging, Vol. 133, 2011, pp. 14- 19.
- [17] D. Sujan, M. V. V. Murthy, K. N. Seetharamu., “*Improved Tri-Layered Interfacial Stress Model With The Effect of Different Temperatures in the Layers*”, Arch Appl. Mech., Vol. 81, 2011, pp. 561 - 568.
- [18] Aswatha, A. N. Vinay, K. S. Naresh, M. V. V. Murthy, K. N. Seetharamu, *Effect of Bond Layer on Tri-material Assembly subjected to Uniform Temperature Change*, Int. Conf. on Computer Aided Engineering (CAE-2013), Department of Mechanical Engineering, IIT Madras, India.